

# Thermal Wightman Functions and Renormalized Stress Tensors in the Rindler Wedge

Valter Moretti <sup>1</sup>, Luciano Vanzo <sup>2</sup>

Dipartimento di Fisica, Università di Trento  
and Istituto Nazionale di Fisica Nucleare,  
Gruppo Collegato di Trento, Italia

July - 1995

## Abstract

The Wightman functions in the Rindler portion of Minkowski space-time are presented for any value of the temperature and for massless spin fields up to  $s = 1$  and the renormalized stress tensor relative to Minkowski vacuum is discussed. A gauge ambiguity in the vector case is pointed out.

PACS number(s): 04.62.+v, 11.10.Wx

## 1 Wightman Functions and Stress Tensors

The Rindler regions can be defined with respect to any spacelike two-plane  $\mathcal{P}$  in Minkowski space-time. We may choose rectangular coordinates  $(x, y, z, t)$  such that the plane is the set  $x = t = 0$ . The Rindler wedge we shall consider, denoted  $W_R$ , will then be defined by the inequality  $x > |t|$ . A global parametrization of  $W_R$  is obtained by setting  $x = \xi \cosh \tau$ ,  $t = \xi \sinh \tau$ , for  $\xi > 0$ , so that  $x^2 - t^2 = \xi^2$ . Thus any line  $\xi = \xi_0$ ,  $y = y_0$ ,  $z = z_0$  will be the trajectory of a uniformly accelerated particle, with proper acceleration  $a = \xi_0^{-1}$  and proper time  $s = a\tau$  along the trajectory. The Minkowski metric will take the form  $ds^2 = -\xi^2 d\tau^2 + d\xi^2 + dx_t^2$ , with  $\xi > 0$  and  $x_t = (y, z)$  standing for the transverse coordinates. The metric admits the timelike Killing field  $K = \partial_\tau$  generating the isometry  $\tau \rightarrow \tau + \tau_0$ . The hypersurface  $\xi = 0$  is an event horizon which bifurcates in the transverse two-plane  $\mathcal{P}$ .

We shall find the thermal Wightman functions in the Rindler region  $W_R$  (the left region  $W_L$  is then covered by reflecting through the wedge, namely by sending  $(t, x, x_t) \rightarrow (-t, -x, x_t)$ ). Hence it is understood that fields quantization in this region is defined by

---

<sup>1</sup>e-mail: [moretti@science.unitn.it](mailto:moretti@science.unitn.it)

<sup>2</sup>e-mail: [vanzo@science.unitn.it](mailto:vanzo@science.unitn.it)

taking the Fock representation over a vacuum  $|F\rangle$  which is invariant under translations in  $\tau$  (it is customary to call  $|F\rangle$  the Fulling vacuum [1, 2, 3, 4] (for an alternative description of accelerated systems, see [5]).

The vacuum Wightman functions for a general field  $\phi_A(x)$  are then defined as the expectation values

$$W_{AB}^+(x, x') = \langle F | \phi_A(x) \phi_B(x') | F \rangle, \quad W_{AB}^-(x, x') = \langle F | \phi_B(x') \phi_A(x) | F \rangle \quad (1)$$

The definition will be the same for other states as well, in particular for the Minkowski vacuum  $|M\rangle$ . The situation will be rather different for a thermal equilibrium state, since then there is no obvious way to compute the expectation values. This is because the partition function for a quantum field is divergent in the infinite volume of the Rindler region. The thermal Wightman functions will then be defined as the periodic or anti-periodic solution of the field equations having the analyticity properties which are demanded by the KMS condition [6, 7]. An independent check will be then to recover the vacuum expectation values in the limit  $\beta \rightarrow \infty$  of zero temperature. For future reference we define the quantity  $\alpha$  by

$$\cosh \alpha = \frac{\xi^2 + \xi'^2 + |x_t - x'_t|^2}{2\xi\xi'}$$

The Weyl and electromagnetic fields will be defined with respect to the natural orthonormal vierbein

$$e_{(0)}^a = \xi^{-1} \delta_0^a, \quad e_{(i)}^a = \delta_i^a, \quad i = 1, 2, 3$$

where  $a, b, c, \dots$  denotes coordinate indices and  $i, j, k, \dots$  anholonomic, or vierbein indices. The thermal Wightman functions at inverse temperature  $\beta$  for a massless field with elicity  $s > 0$  will be denoted by  $W^{(s)\pm}(\beta|x, x')$  and simply by  $W^\pm(\beta|x, x')$  in the spin zero case. We give them first and then we discuss how they were obtained. They are given as follows: a) the scalar  $s = 0$  field

$$W^+(\beta|x, x') = \frac{1}{4\pi\beta\xi\xi' \sinh \alpha} \left[ \frac{\sinh \frac{2\pi\alpha}{\beta}}{\cosh \frac{2\pi}{\beta}\alpha - \cosh \frac{2\pi}{\beta}(\tau - \tau' - i\varepsilon)} \right] \quad (2)$$

and  $W^-(\beta|x, x') = [W^+(\beta|x, x')]^*$ . These are manifestly periodic in imaginary time with period  $\beta$ . To our knowledge, this result was first obtained by J.S.Dowker [8, 9]. The zero temperature limit is

$$W^+(x, x') = \langle F | \phi(x) \phi(x') | F \rangle = -\frac{1}{4\pi^2} \frac{\alpha}{\xi\xi' \sinh \alpha} \frac{1}{(\tau - \tau' - i\varepsilon)^2 - \alpha^2} \quad (3)$$

and  $W^-(x, x') = \langle F | \phi(x') \phi(x) | F \rangle = [W^+(x, x')]^*$ . Note that these functions are vacuum expectation values in the Fulling state. The thermal function can be obtained by the sum over images

$$W^\pm(\beta|x, x') = -\frac{1}{4\pi^2} \frac{\alpha}{\xi\xi' \sinh \alpha} \sum_{n=-\infty}^{\infty} \frac{1}{(\tau - \tau' \mp i\varepsilon - in\beta)^2 - \alpha^2} \quad (4)$$

The value  $\beta = 2\pi$  is distinguished by the property

$$\begin{aligned} W^+(2\pi|x, x') &= \frac{1}{4\pi^2} \frac{1}{\xi^2 + \xi'^2 + |x_t - x'_t|^2 - 2\xi\xi' \cosh(\tau - \tau' - i\varepsilon)} \\ &= -\frac{1}{4\pi^2} \frac{1}{(t - t' - i\varepsilon)^2 - (x - x')^2 - |x_t - x'_t|^2} \end{aligned} \quad (5)$$

which is just the Wightman function which characterizes the Minkowski vacuum state. This means that this vacuum is a KMS state with respect to  $\tau$ -translation [10, 2, 3, 11, 8, 12], with inverse temperature  $\beta = 2\pi$ . The thermal stress tensor relative to the Minkowski vacuum is [13, 14, 15]

$$T^{ab} = \frac{1}{1440\pi^2\xi^4} \left[ \left( \frac{2\pi}{\beta} \right)^4 - 1 \right] [4v^a v^b + g^{ab}] \quad (6)$$

where  $v^a = K^a/\sqrt{-K^2}$ ,  $K = \partial_\tau$  being the Killing vector field of the Rindler region. The zero temperature stress tensor reduces to the one calculated in Ref.[16].

b) Weyl  $s = 1/2$  fermions.

There are two irreducible representations of the Dirac algebra. Denoting by  $\sigma$  the Pauli matrices, these are given by  $\sigma^i = (e, \sigma)$  and  $\tilde{\sigma}^i = (e, -\sigma)$ , where  $e$  is the unit. In the tilde representation we find

$$W^{(1/2)+}(\beta|x, x') = i\tilde{\sigma}^a \tilde{\nabla}_a F_\beta^+(x, x')$$

$$W^{(1/2)-}(\beta|x, x') = -i\tilde{\sigma}^a \tilde{\nabla}_a F_\beta^-(x, x')$$

where

$$\begin{aligned} F_\beta^+(x, x') &= \frac{e}{4\pi\beta\xi\xi' \sinh \frac{\alpha}{2}} \left[ \frac{\sinh\left(\frac{\pi}{\beta}\alpha\right) \cosh\left[\frac{\pi}{\beta}(\tau - \tau')\right]}{\cosh\left(\frac{2\pi}{\beta}\alpha\right) - \cosh\left[\frac{2\pi}{\beta}(\tau - \tau' - i\varepsilon)\right]} \right] \\ &+ \frac{\sigma_1}{4\pi\beta\xi\xi' \cosh \frac{\alpha}{2}} \left[ \frac{\cosh\left(\frac{\pi}{\beta}\alpha\right) \sinh\left[\frac{\pi}{\beta}(\tau - \tau')\right]}{\cosh\left(\frac{2\pi}{\beta}\alpha\right) - \cosh\left[\frac{2\pi}{\beta}(\tau - \tau' - i\varepsilon)\right]} \right] \end{aligned} \quad (7)$$

These are manifestly anti-periodic in imaginary time with period  $\beta$ , in accord with the KMS condition. The zero temperature limit is

$$W^{(1/2)+}(x, x') = \langle F|\psi(x)\psi^\dagger(x')|F \rangle = i\tilde{\sigma}^a \tilde{\nabla}_a F^+(x, x')$$

$$W^{(1/2)-}(x, x') = -\langle F|\psi^\dagger(x')\psi(x)|F \rangle = -i\tilde{\sigma}^a \tilde{\nabla}_a F^-(x, x')$$

where

$$\begin{aligned} F^+(x, x') &= -\frac{e}{8\pi^2\xi\xi' \sinh \frac{\alpha}{2}} \left[ \frac{\alpha}{(\tau - \tau' - i\varepsilon)^2 - \alpha^2} \right] - \\ &- \frac{\sigma_1}{8\pi^2\xi\xi' \cosh \frac{\alpha}{2}} \left[ \frac{\tau - \tau'}{(\tau - \tau' - i\varepsilon)^2 - \alpha^2} \right] \end{aligned} \quad (8)$$

and  $F^- = (F^+)^*$ . Note that these functions are vacuum expectation values in the Fulling state  $|F\rangle$ . The sum over images with alternating signs gives again the above thermal functions. The special value  $\beta = 2\pi$  is distinguished, since then

$$W^{(1/2)\pm}(2\pi|x, x') = \pm i\tilde{\sigma}^a \nabla_a W^\pm(x, x')$$

where  $W^\pm(x, x')$  are the Wightman function for the massless scalar field. These are just the Wightman functions for neutrinos in the Minkowski fermion vacuum, relative to a boosted tetrad. The stress tensor relative to the Minkowski vacuum has the perfect fluid form[17, 15]

$$T^{ab} = \frac{1}{11520\pi^2\xi^4} \left[ 7 \left( \frac{2\pi}{\beta} \right)^4 + 10 \left( \frac{2\pi}{\beta} \right)^2 - 17 \right] [4v^a v^b + g^{ab}] \quad (9)$$

The zero temperature tensor was calculated in Ref.[18] (see also Ref.[19]).

c) The electromagnetic  $s = 1$  field.

The tetrad components of the Wightman functions  $W_{ij'}^{(1)\pm}(\beta|x, x')$  will be given in the Feynman gauge  $\nabla_a A^a = 0$ , where a prime over the indices means that the function is a bivector at  $x$  and  $x'$  respectively. They are given by the equations

$$W_{00'}^{(1)+}(\beta|x, x') = \frac{-1}{4\pi\beta\xi\xi' \sinh \alpha} \frac{\cosh\left(\frac{2\pi}{\beta}(\tau - \tau')\right) \sinh \alpha + \sinh\left(\frac{2\pi}{\beta} - 1\right) \alpha}{\cosh\left(\frac{2\pi}{\beta}\alpha\right) - \cosh\left(\frac{2\pi}{\beta}(\tau - \tau' - i\varepsilon)\right)} \quad (10)$$

$$W_{11'}^{(1)+}(\beta|x, x') = -W_{00'}^{(1)+}(\beta|x, x') \quad (11)$$

$$W_{10'}^{(1)+}(\beta|x, x') = \frac{-1}{4\pi\beta\xi\xi'} \frac{\sinh\left(\frac{2\pi}{\beta}(\tau - \tau')\right)}{\cosh\left(\frac{2\pi}{\beta}\alpha\right) - \cosh\left(\frac{2\pi}{\beta}(\tau - \tau' - i\varepsilon)\right)} \quad (12)$$

$$W_{01'}^{(1)+}(\beta|x, x') = -W_{10'}^{(1)+}(\beta|x, x') \quad (13)$$

$$W_{22'}^{(1)+}(\beta|x, x') = W_{33'}^{(1)+}(\beta|x, x') = W^+(\beta|x, x') \quad (14)$$

where  $W^+(\beta|x, x')$  is the scalar Wightman function, Eq. (2). The periodicity in imaginary time is again evident. In Ref.[15] the Green functions for any spin around a cosmic string have been given in the  $(j, 0)$  representation of the Lorentz group. For  $s = 1$  elicity fields these are Green functions for the fields  $\mathbf{E} \pm i\mathbf{H}$ , and subtle questions of gauge invariance were consequently avoided. The zero temperature limit is

$$W_{00'}^{(1)+}(x, x') = -W_{11'}^{(1)+}(x, x') = \frac{-1}{4\pi^2\xi\xi'} \frac{\alpha \coth \alpha}{\alpha^2 - (\tau - \tau' - i\varepsilon)^2} \quad (15)$$

$$W_{10'}^{(1)+}(x, x') = -W_{01'}^{(1)+}(x, x') = \frac{-1}{4\pi^2\xi\xi'} \frac{\tau - \tau'}{\alpha^2 - (\tau - \tau' - i\varepsilon)^2} \quad (16)$$

$$W_{22'}^{(1)+}(x, x') = W_{33'}^{(1)+}(x, x') = W^+(x, x') \quad (17)$$

where  $W^+(x, x')$  is the Wightman function for the scalar field, Eq. (2). They are expectation values in the Fulling vacuum state, which satisfies  $\nabla^a A_a^{(+)}|F\rangle = 0$ , namely

$$W_{ij'}^{(1)+}(x, x') = \langle F|A_i(x)A_j(x')|F\rangle$$

The verification of this statement from canonical quantization is rather messy, due to an apparent divergence in the integral representation of the Rindler Wightman functions. This representation also appeared in Ref.[20], where the Fulling-Davies-Unruh thermal bath was shown to be exactly the bremsstrahlung radiation emitted by a uniformly accelerated charge. The value  $\beta = 2\pi$  is distinguished since then

$$W_{ij'}^{(1)\pm}(2\pi|x, x') = g_{ij}W^\pm(x, x')$$

where  $W^\pm(x, x')$  is the scalar Wightman function. This is just the Wightman function in the Feynman gauge of Minkowski vacuum relative to a boosted tetrad. The Wightman functions obey the Ward identity. In the Feynman gauge this identity states that

$$\nabla^a W_{ab'}^{(1)\pm} + \nabla_{b'} W_{gh}^\pm = 0 \quad (18)$$

where  $W_{gh}^\pm$  is the Wightman function for the ghost fields  $\eta_1(x)$ ,  $\bar{\eta}_2(x)$ . This is actually equal to the scalar Wightman function because the ghosts equations of motion are  $\square\eta_{1,2}(x) = 0$ . Though uncoupled to the electromagnetic field, their presence is essential in the finite temperature theory[21]. The stress tensor relative to the Minkowski vacuum has the perfect fluid form[15]

$$T^{ab} = \frac{1}{720\pi^2\xi^4} \left[ \left( \frac{2\pi}{\beta} \right)^4 + 10 \left( \frac{2\pi}{\beta} \right)^2 - 11 \right] [4v^a v^b + g^{ab}] \quad (19)$$

## 2 Discussion

The previous *non* thermal Wightman functions were obtained from canonical quantization by calculating explicitly the integral representations of the field operators vacuum expectation values. Conversely, in the case of finite  $\beta$ , we used different methods depending on the value of the spin. In fact, for scalar and spinorial fields one can employ the method of images, obtaining the corresponding non thermal Wightman functions written above. One can also implement the canonical formalism, within the Feynmann gauge, in the photon case. Then the integral representation of  $W_{00'}^{(1)}$  and  $W_{11'}^{(1)}$  follows from the normal modes decomposition of the field operator  $A_a$ , in the form

$$\begin{aligned} W_{00'}^{(1)}(x, x') &= -W_{11'}^{(1)}(x, x') = \langle F|A_0(x)A_{0'}(x')|F\rangle = -\langle F|A_1(x)A_{1'}(x')|F\rangle = \\ &= \frac{1}{4\pi^4} \int_{\mathbb{R}^2} dk_t \int_0^{+\infty} d\omega \frac{\sinh \pi\omega}{k_\perp^2} D K_{i\omega}(|k_t|\xi) K_{i\omega}(|k_t|\xi') e^{ik_t \cdot (x_t - x'_t)} e^{-i\omega(\tau - \tau' - i\varepsilon)}, \end{aligned}$$

where  $K_{i\omega}(x)$  is the well-known Mc Donald function of imaginary index and the operator  $D$  is defined as

$$D = \frac{1}{\xi\xi'} [-\partial_\tau \partial_{\tau'} + \xi \partial_\xi \xi' \partial_{\xi'}].$$

One can solve the above integral (and the more trivial integral representations corresponding to the remaining components) obtaining just the non thermal Wightman functions in Eq.s (15), (16), (17). Then, the sum over images method produced the following result quite trivially

$$\begin{aligned}\tilde{W}_{00'}^{(1)}(\beta|x, x') &= -\tilde{W}_{11'}^{(1)}(\beta|x, x') = W_{00'}^{(1)}(\beta|x, x') - \frac{1}{4\pi\beta\xi\xi'}, \\ \tilde{W}_{ab'}^{(1)}(\beta|x, x') &= W_{ab'}^{(1)}(\beta|x, x') \text{ in all the remaining cases,}\end{aligned}\tag{20}$$

where  $\tilde{W}_{ab'}^{(1)}(\beta|x, x')$  indicates the sum over images result arising from  $W_{ab'}^{(1)}(x, x')$  and the functions  $W_{ab'}^{(1)}(\beta|x, x')$  were defined by Eq.(10)-(14). Surprisingly then, due to the anomalous term in Eq.(20), the periodicity sum of the zero temperature Wightman functions so obtained fails to reduce to the Minkowski Wightmann functions when  $\beta = 2\pi$  and thus fails to reproduce exactly the finite temperature result, since the thermal properties of the Minkowski vacuum relative to Rindler time translations can be established by independent arguments [10] and even in a model independent and rigorous way [22]. We also observe that the sum over images behaves badly as  $x_t \rightarrow \infty$  because  $\tilde{W}_{11'}^{(1)}(\beta)$  and  $\tilde{W}_{00'}^{(1)}(\beta)$  do not vanish there as one might expect (in the case of  $\beta = 2\pi$  at least). However the correct non thermal Wightman functions are reached in the limit  $\beta \rightarrow +\infty$ .

A closer scrutiny of the situation reveals that the responsibility of the failure is due to the sector of the photon Fock space containing Rindler states with negative norm. In fact, few calculations prove that the anomalous term  $\Delta_{ab'}(\beta) = \tilde{W}_{ab'}^{(1)}(\beta) - W_{ab'}^{(1)}(\beta)$  vanishes when this acts as a three-distribution on three-smearred solutions of vectorial Klein-Gordon equation built up with physical modes only (and having compact support on the Rindler Cauchy surfaces, for example) <sup>3</sup>.

Nevertheless, the result obtained by periodicity sum obeys both the wave equation and the Ward identities because  $\Delta_{ab'}(\beta)$  is a solution of the vectorial Klein Gordon equation having vanishing divergence.

On the other hand, the Wightman functions for the field strength (thus containing no negative norm states), can be obtained by periodicity summing over the zero temperature functions  $\langle F|F_{ab}(x)F_{a'b'}(x')|F \rangle$  because the anomalous term produces a vanishing field strength. We conclude that, dealing with physical quantities, the anomalous term has no consequences because it represents a gauge ambiguity.

Therefore it is possible to drop completely the anomalous term  $\Delta_{ab'}(\beta)$  in the result obtained by summing over images, giving just the Wightman functions appearing in Eq.s (10)-(14) which reduce to the Minkowski Wightman functions when  $\beta = 2\pi$ .

Independently on the method of images, the Wightman functions of Eq.s (10)-(14) can be obtained extending Dowker technique to handle the scalar Green function on a conical space[23]. The Rindler metric tensor in euclidean time with  $\beta$  periodicity just represents a conical space of the form  $\mathcal{C}_\beta \times \mathbb{R}^2$ ,  $\mathcal{C}_\beta$  being a two dimensional cone with deficit angle  $\gamma = 2\pi - \beta$ . The Green function of a vector field can then be obtained in closed form on the cone after which by analytic continuation back to real time one gets precisely Eq.s (10)-(14).

It is interesting to observe that, differently from the method of images, this euclidean approach forces automatically the Wightman functions to behave correctly at infinity. A

---

<sup>3</sup>Using a four-smearred formalism, the anomalous term vanishes acting as a distribution on conserved currents defined into the *open* Rindler wedge.

complete calculation for photons and gravitons was also presented in Ref.[24], for the case of a cosmic string background in which the conical singularity was rounded off. Upon translating their results to Rindler space, one finds complete agreement. To conclude, great care is necessary to deal with gauge fields in accelerated frames and in covariant gauges. Related difficulties have also been encountered in [25] and precisely in the same context.

The thermal stress tensor was obtained by the cited authors using the point splitting procedure. Here we give an independent argument which is based on the old observation[26] that the manifold  $\mathcal{M} = \mathbb{R} \times H^3$ , with the natural product metric, is conformal to Rindler space. Here  $H^3$  is the hyperbolic three space carrying a metric with constant negative curvature (an extensive discussion of conformally invariant quantum field theory in hyperbolic universes has also been given in Ref.[27, 13]).

The one-loop partition function (per unit volume) for a thermal state in  $\mathcal{M}$  will be determined by the density of one-particle states in  $H^3$ , denoted  $\nu^{(s)}(\omega)$  for a spin  $s$  field. Indeed

$$\log Z^{(s)}(\beta|\xi) = \xi^{-3} \int_0^\infty \log(1 \pm e^{\beta\omega}) \nu^{(s)}(\omega) d\omega - \beta U \quad (21)$$

the factor  $\xi^{-3}$  coming from the optical space volume element.  $U$  is the vacuum energy density, the only quantity that needs a renormalization prescription in this contest. The conformal transformation back to Rindler only adds a  $\beta$ -linear term[28, 29], which may be absorbed into the definition of  $U$ . The density of states is thus the crucial quantity. In  $H^N$  and for the Laplace-Beltrami operator, it has long been known by mathematician where it is known as the Harish-Chandra or Plancherel measure (see Ref.[30] and Ref.s therein for a detailed account). In  $H^3$  it is

$$\nu^{(0)}(\omega) = \frac{\omega^2}{2\pi^2} \quad (22)$$

$$\nu^{(1/2)} = \frac{(\omega^2 + 1/4)}{2\pi^2} \quad (23)$$

$$\nu^{(1)} = \frac{(\omega^2 + 1)}{\pi^2} \quad (24)$$

where the  $s = 1$  case holds for transverse vector fields in  $H^3$  (this corresponds to the Coulomb choice of gauge in  $\mathbb{R} \times H^3$ ). The partition function is now easily computed from Eq. (21). We give the details for  $s = 0$  only, the other cases being similar. We obtain

$$\log Z(\beta|\xi) = \frac{\pi^2}{90\xi^3} \beta^{-3} - \beta_T U(\Lambda), \quad \beta_T = \xi\beta \quad (25)$$

where  $\beta_T$  is the Tolman inverse temperature and

$$U(\Lambda) = \frac{1}{4\pi^2\xi^4} \int_0^\Lambda \omega^3 d\omega$$

ie the regularized vacuum energy density. The linear term will not affects the entropy density while the energy density must vanishes at  $\beta = 2\pi$ , since this would correspond to the scalar vacuum in Minkowski space-time, whose energy density is defined to be zero

in order to realize the Poincaré symmetry. The zero point of entropy will also vanishes at  $\beta = \infty$  since the Fulling vacuum is a pure states. Once the zero point of entropy and energy density have been fixed, there is no further room left and all the thermodynamics densities are fixed. Thus we get the renormalized energy density and pressure

$$u(\beta) = 3p = \frac{1}{480\pi^2\xi^4} \left[ \left( \frac{2\pi}{\beta} \right)^4 - 1 \right] \quad (26)$$

the entropy density

$$s(\beta) = \frac{4\pi^2}{90\xi^3} \beta^{-3} \quad (27)$$

and the free energy density

$$f(\beta) = -\frac{\pi^2}{90\xi^4} [\beta^{-4} + 3(2\pi)^{-4}] \quad (28)$$

The cut-off dependence is now disappeared. Why should not we define the zero of entropy at  $\beta = 2\pi$  which also is a pure state, namely the Minkowski vacuum? The reason is a well known consequence of quantum theory, first discovered by von Neumann[31], that a subsystem of a system in a pure state may has a non zero entropy if only the subsystem is being observed. Now while it is true that at  $\beta = 2\pi$  we are computing quantities in the Minkowski vacuum, we are actually probing only the right hand side Rindler wedge since the field operators from which the above results were derived were restricted over there. Notice that the zero point free energy is equal to the zero point energy and that the Gibbs relation  $Ts = u+p$  gets modified to  $Ts = (u-u_0)+(p-p_0)$ , in accord with thermodynamic. The total entropy is infinite even when  $\beta = 2\pi$ , this being the normal behaviour which is associated with acceleration horizons. It has been shown that the thermodynamic entropy as given above is the same as the entanglement entropy[11, 32, 22], in accord with von Neumann ideas. Eq. (6) for the stress tensor can now be derived by noting that the energy density and pressure must be the eigenvalues of the stress tensor in an orthonormal vierbein.

## References

- [1] S.A. Fulling. Phys. Rev. **D7**, 2850 (1973).
- [2] S.A. Fulling. J. Phys. A: Math. Gen. **10**, 917 (1977).
- [3] W.G. Unruh. Phys. Rev. **D14**, 870 (1976).
- [4] D.G. Boulware. Phys. Rev. **D11**, 1404 (1975).
- [5] N. Sanchez and B.F. Whiting. Phys. Rev. **D34**, 1056 (1986).
- [6] R. Haag, N.M. Hugenholtz and M. Winnink. Commun. Math. Phys. **5**, 215 (1967)
- [7] S.A. Fulling and S.N.M. Ruijsenaars. Phys. Rep. **152**, 135 (1987).
- [8] J.S. Dowker. J. Phys. **A10**, 115 (1977).



- [9] J.S. Dowker. Phys. Rev. **18**, 1856 (1978).
- [10] R. Haag, H. Narnhofer and U. Stein. Commun. Math. Phys. **94**, 219 (1984).
- [11] W. Israel. Phys. Lett. **A57**, 107 (1976).
- [12] P.C.W. Davies. J. Phys. **A8**, 609 (1975).
- [13] M.R. Brown, A.C. Ottewill and D. Page. Phys. Rev. **D33**, 2840 (1986).
- [14] M.R. Brown and A.C. Ottewill. Phys. Rev. **D31**, 2514 (1985).
- [15] J.S. Dowker. Phys. Rev. **D36**, 3742 (1987).
- [16] P. Candelas and D. Deutsch. Proc. R. Soc. Lond **A254**, 79 (1977).
- [17] V.P. Frolov and E.M. Serebriany. Phys. Rev. **D35**, 3779 (1987).
- [18] P. Candelas and D. Deutsch. Proc. R. Soc. Lond **A362**, 78 (1978).
- [19] W. Troost and H. van Dam. Nucl. Phys. **B159**, 442 (1979).
- [20] A. Higuchi, G. E. Matsas and D. Sudarsky. Phys. Rev. **D46**, 3450 (1992).
- [21] C. Bernard. Phys. Rev. **D9**, 3312 (1974).
- [22] G. L. Sewell. Ann. Phys. **141**, 201 (1982).
- [23] J. S. Dowker. Phys. Rev. **D36**, 3095 (1987).
- [24] B. Allen, J. G. Mc Laughlin and A. C. Ottewill. Phys. Rev. **D45**, 4486 (1992).
- [25] D. Kabat. Nucl. Phys. **B453**, 281 (1995).
- [26] P. Candelas and J.S. Dowker. Phys. Rev. **D19**, 2902 (1979).
- [27] T.S. Bunch. Phys. Rev. **D18**, 1844 (1978).
- [28] J. S. Dowker. Phys. Rev. **D33**, 3150 (1986).
- [29] J.S. Dowker and G. Kennedy. J. Phys. **A11**, 895 (1978).
- [30] R. Camporesi. Phys. Rep. **196**, 1 (1990).
- [31] J. von Neumann. *Mathematical Foundations of Quantum Mechanics*. Princeton University Press, (1955).
- [32] D. Kabat and M. J. Strassler. Phys. Lett. **B329**, 46 (1994).